		$g^{1/\gamma_4}\Gamma_{41}$ FOR TAILORING		
		SHOCK MACH NO. ~ M_s		
γ_4	γ_1	3	5	∞
1.67	1.2	24	46	83
	1.4	13	18	23
	1.67	8	10	12
1.4	1.2	27	53	99
	1.4	13	20	27
	1.67	8	10	13
1.2	1.2	31	70	136
	1.4	15	24	34
	1.67	8	10	13

Fig. 2 Tailored-interface conditions for shock tubes (ideal gases)

The region of tailored-interface operation, where the shock wave reflected from the driven-tube end produces no reflected waves on interacting with the interface, also is shown. For shock waves sufficiently strong that $M_s^2 \gg (\gamma_1 + 1)/(\gamma_1 - 1)$ (i.e., limiting density ratio for ideal gas), tailoring occurs at one point only on the $M_s/(gP_{41})^{1/2}$ vs $g^{1/\gamma_4}\Gamma_{41}$ curve. The range of $g^{1/\gamma_4}\Gamma_{41}$ over which tailoring occurs for $3 \leq M_s \leq \infty$ is quite narrow. Conditions for tailoring are summarized in Fig. 2.

The use of these reduced variables simplifies the analysis of more complex shock-tube configurations. In the case of the buffered shock tube,[3] for example, similar approximations give the result that the final shock Mach number attained for given initial states of the driver and driven gases depends only on the initial driver-buffer gas density ratio. Previous numerical results for the buffered shock tube which show the effects of varying buffer-gas pressure and molecular weight[3] can be correlated conveniently in terms of this density ratio.

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Electromagnetic Torques Operating on Satellites Using Snap Reactor Power Systems

J. H. Bick*
North American Aviation Science Center, Canoga Park, Calif.

THE satellites that use Snap nuclear reactor systems for auxiliary power are unique in both the size of the permanent magnet carried and the amount of electric current generated while in orbit. The interaction of the electric and magnetic fields of the satellites with the geomagnetic field will produce torques that may affect the satellite attitude. It is the purpose of this note to express these electromagnetic torques as a function of time in order that they may be used

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* Member of the Technical Staff.

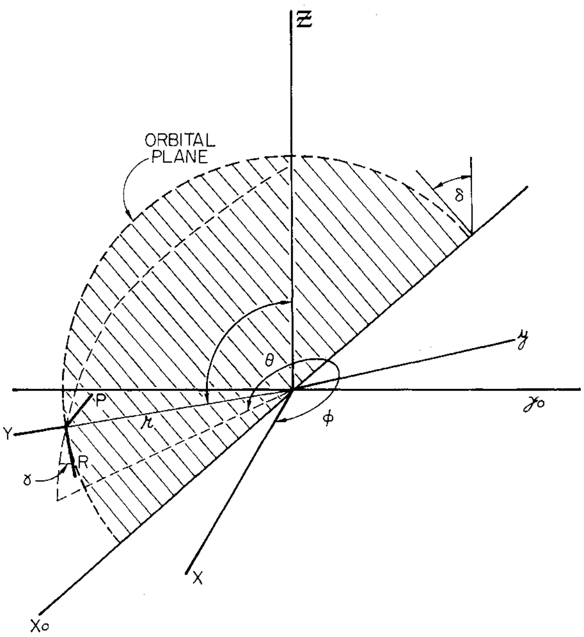

Fig. 1 Coordinate system used in torque analysis

as an inhomogeneous forcing function in the dynamic attitude control equations.

The geomagnetic field may be derived from a potential, $V(r,\theta,\varphi)$ expressed in powers of orbit radius and normalized associated Legendre polynomials.[1] The components of the field at the location of the satellite are

$$B_r = \partial V/\partial r \tag{1}$$

$$B_\theta = (1/r)(\partial V/\partial \theta) \tag{2}$$

$$B_\varphi = -(1/r\,\sin\theta)(\partial V/\partial\varphi) \tag{3}$$

where r, θ, and φ are conventional spherical coordinates centered in the earth, θ being measured from the North Pole. The more terms taken in the series expansions for the field components, the higher the degree of accuracy that will be obtained. The magnetic moment of the satellite has components in the roll, pitch, and yaw directions, where the yaw axis is parallel to the local vertical, the roll axis is in the orbital plane and perpendicular to the yaw, and the pitch axis forms the third axis of a right-handed Cartesian set, permuted in the order roll, pitch, yaw (see Fig. 1). Note that, if the orbital plane contains the line connecting the geographic poles, the roll, yaw, and pitch components of the satellite magnetic moment vector are, respectively, parallel to B_θ, B_r, and B_φ.

The perturbing torque is the vector product of the satellite and geomagnetic fields. If one lets γ be the positive angle measured from the positive roll axis to the tangent to the circle, r = const at the satellite (see Fig. 1), and defines the components of the satellite field as M_R, M_P, and M_Y along the roll, pitch, and yaw axes, then the torque components about these axes are

$$T_R = M_Y(B_\varphi \cos\gamma - B_\theta \sin\gamma) - M_P B_r \tag{4}$$

$$T_P = M_R B_r - M_Y(B_\varphi \cos\gamma - B_\theta \sin\gamma) \tag{5}$$

$$T_Y = M_P(B_\theta \cos\gamma + B_\varphi \sin\gamma) - M_R(B_\varphi \cos\gamma - B_\theta \sin\gamma) \tag{6}$$

expressed in the roll, pitch, yaw coordinate system.

Having found the components of the torque vector, it is necessary to express the orbital radius and the angles γ, θ, and

[1] Johnson, F. S., *Satellite Environment Handbook* (Stanford University Press, Stanford, Calif., 1961), p. 127.

φ as functions of time in order that the torques can be used in the attitude control equations. This problem will be solved for a circular orbit. Consider two sets of Cartesian coordinates, one (x,y,z) fixed in the earth, and one (x_0,y_0,z_0) fixed with respect to the orbital plane. Without loss of generality, it can be assumed that the orbital plane contains the x_0 axis. In the (x_0,y_0,z) system of coordinates,

$$z = r \cos\delta \cos\omega_1 t \tag{7}$$

$$x_0 = r \sin\omega_1 t \tag{8}$$

$$y_0 = r \sin\delta \cos\omega_1 t \tag{9}$$

where δ is the angle that the orbital plane makes with the z axis and ω_1 is the frequency of the satellite orbit. Transforming to the x,y,z coordinates by means of

$$x = x_0 \cos\omega_2 t + y_0 \sin\omega_2 t \tag{10}$$

$$y = -x_0 \sin\omega_2 t + y_0 \cos\omega_2 t \tag{11}$$

where ω_2 is the frequency of the earth's rotation, one obtains

$$\begin{aligned} \theta &= \cos^{-1}(z/r) \\ &= \cos^{-1}[\cos\delta \cos\omega_1 t] \end{aligned} \tag{12}$$

$$\begin{aligned} \varphi &= \tan^{-1}(y/x) \\ &= \tan^{-1}\left[\frac{-\sin\omega_1 t \sin\omega_2 t + \sin\delta \cos\omega_1 t \cos\omega_2 t}{\sin\omega_1 t \cos\omega_2 t + \sin\delta \cos\omega_1 t \sin\omega_2 t}\right] \end{aligned} \tag{13}$$

The angle between the roll axis and the meridians of longitude is given by

$$\gamma = \tan^{-1}(\tan\delta \csc\omega_1 t) \tag{14}$$

Equations (1–3 and 12–14), when substituted into Eqs. (4–6), will give the electromagnetic torques operating on the satellite as explicit functions of time. For a polar ($\delta = 0$) orbit, Eqs. (12–14) reduce to $\theta = \omega_1 t$, $\varphi = -\omega_2 t$, and $\gamma = 0$.

Present designs of satellites bearing Snap reactor power systems call for a cylindrical configuration having approximately equal moments of inertia about pitch and roll axes, and a much smaller moment about the yaw axis. In this case, it is necessary to stabilize the satellite about the yaw axis with some arrangement of gyroscopes, and it is therefore desirable to minimize the magnetic torques about the yaw axis.

Since the azimuthal component of the geomagnetic field B_φ is generally small when compared with B_θ or B_r, Eq. (6) shows that T_Y can be minimized by locating the satellite in a polar orbit ($\gamma = 0$) and by aligning the satellite magnetic moment along the yaw axis. If this latter requirement cannot be accomplished, in no case should the satellite moments be aligned with the pitch axis.

Large-Scale Gap Test: Comparison of Tetryl and Pentolite Donors

John Toscano,* Irving Jaffe,† and George Roberson‡
U. S. Naval Ordnance Laboratory, White Oak, Silver Spring, Md.

In the large-scale shock sensitivity test, it is assumed that the pressure amplitude at the 50% point is an intrinsic property of a propellant tested under standardized conditions, provided that the shape of the pressure pulse is defined by the amplitude. In order to examine this proviso and to study the adequacy of pressure amplitude as a measure of shock sensitivity, a second calibration for the gap test was made with a pentolite donor replacing the tetryl donor of the standardized test.

THE large-scale shock sensitivity test (gap test) was calibrated originally at this laboratory with a tetryl donor[1, 2] to interpret the 50% point gap in terms of absolute pressure. The pressure amplitude at the 50% point, assuming the shape of the pressure pulse to be defined by the amplitude, should be an intrinsic property of a propellant tested under standardized conditions and should be reproducible regardless of the donor used. To study the adequacy of 50% pressure as measure of shock sensitivity, a standard pentolite donor was made and used in a second calibration. This donor also was used to determine the 50% point of various substances; the pressures obtained at the 50% point were compared to those obtained with the standard tetryl donor.

Experimental Method

The ingredients of pentolite, 50% trinitrotoluene (TNT) and 50% pentaerythrite tetranitrate (PETN), were prepared according to the joint Army-Navy specification.[3, 4] In addition, careful control of the particle size, mixing process, and density of the pellets was exercised. A no. 70 and a no. 100 sieve (U. S. Standard Sieve Series—ASTM specification) were used to obtain particle sizes of the PETN and TNT ranging from 150 to 210 μ. One thousand grams of each ingredient were then added to a V-blender and dry blended for 1 hr to insure a homogeneous mixture. The pentolite

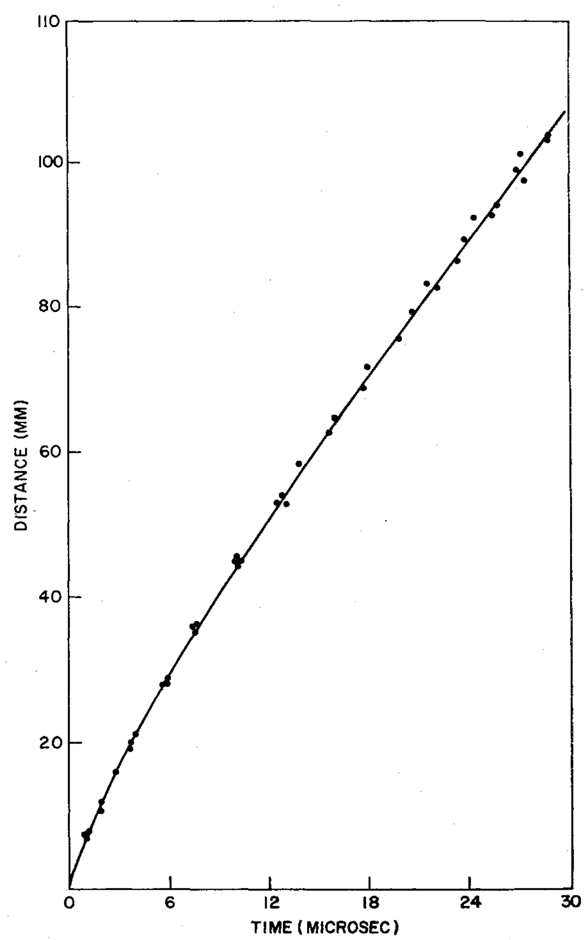

Fig. 1 Shock in Plexiglas

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* Chemical Engineer, Physical Chemistry Division.
† Supervisory Chemist, Physical Chemistry Division.
‡ Physical Science Technican, Physical Chemistry Division.